Lesson 5.1 (F'11)

Objectives: To review the laws of exponents.
To graph exponential functions, finding the horizontal asymptotes.
To work exponential growth and decay problems.
To introduce the number e.
To transform the graphs of exponential functions.

Laws of Exponents

If m, n, a and b are real numbers a > 0 and b > 0, then

1. $b^n \cdot b^m = b^{n+m}$
2. $(b^n)^m = b^{nm}$
3. $(ab)^n = a^n \cdot b^n$
4. $\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$
5. $\frac{b^n}{b^m} = b^{n-m}$
6. $1^m = 1$
7. $\left(\frac{a}{b}\right)^{-n} = \left(\frac{b}{a}\right)^n$
8. $b^0 = 1$

9. If a is a real number $a \neq 0$ and $a^x = a^y$, then x = y

Solve for x.

1. $3^x = 27$
2. $4^x = 8$

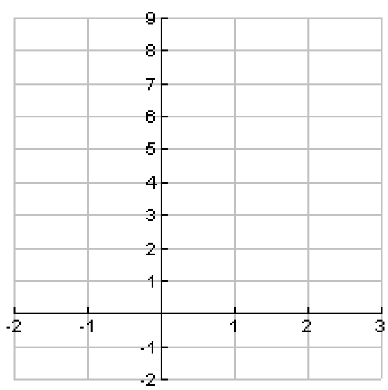
3. $3^x = \frac{1}{9}$

4. $3^x = \sqrt{27}$

5. $64^x = 4$

6. $125^x = 25$

EXAMPLE: Graph $y = 3^x$. State the domain and range. Let’s begin by making a table. Note the horizontal asymptote.



x	y
-2	
-1	
0	
1	
2	

Asymptote__________

Note: The domain is all real numbers, the range is $y > 0$. The horizontal asymptote is the x axis.

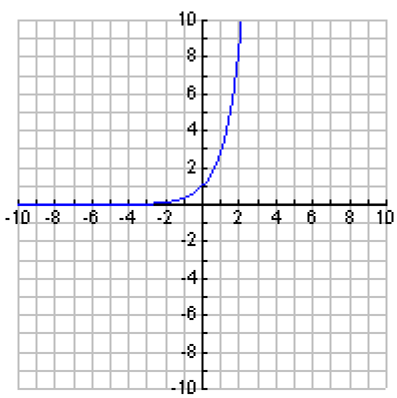
Exponential Function

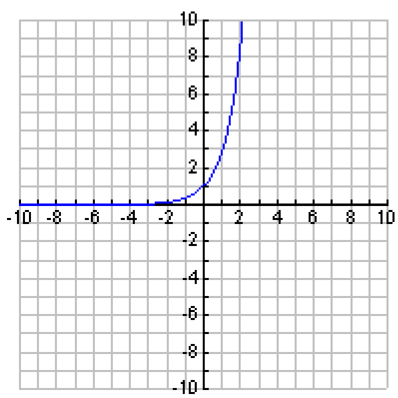
If b is a positive real number, $b \neq 1$, then the function $f(x) = b^x$ is an exponential function. The constant b is called the *base* of the function and the variable x is the *exponent.*

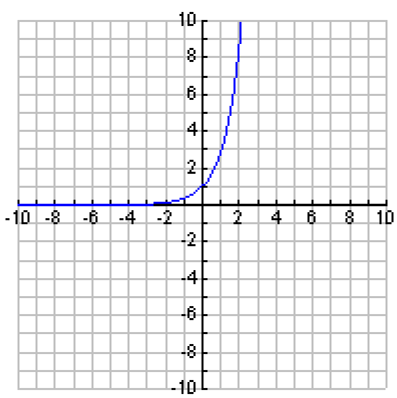
Transformations of Graphs of Exponential Functions

Graph y = 3^x now 1. y = 3^{-x} 2. y = 3^{x+1} -2 3. y = -3^x

Note y = 3^x is already graphed on each graph.







EXAMPLE: Explain how the graph of :

1. $y = 3^{x+4}$ 2. y = $3^x - 2$ 3. y = $3^{x+4} + 2$ compare to y = 3^x

Exponential Growth Function

Equation: $y = a(b^x)$ $b > 1$, $a > 0$
x-intercept: none
y-intercept: $(0, a)$
Domain: All real numbers
Range: All real numbers $y > 0$
Horizontal asymptote: x – axis (the line $y = 0$)
Shape: Increasing on domain and concave up

EXAMPLE: #42 An antique table increases in value according to the function $v(x) = 850(1.04^x)$ dollars, where x is the number of years after 1990.

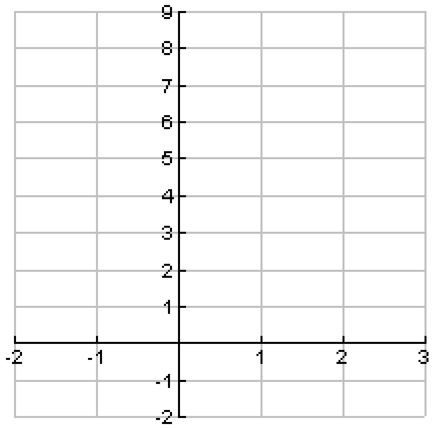
a. How much was the table worth in 1990?

b. If the pattern indicated by the function remains valid, what was the value of the table in 2005?

c. Use a table or graph to estimate the year when this table will reach double its 1990 value.

Exponential Decay

Graph $y = \left(\frac{1}{3}\right)^{x} = 3^{-x}$



x	y
-2	
-1	
0	
1	
2	

Exponential Decay Function

Equation: $y = a(b^{-x})$, $b > 1$, $a > 0$
x-intercept: none
y-intercept: $(0,a)$
Domain: All real numbers
Range: All real numbers $y > 0$
Horizontal asymptote: x- axis (the line $y = 0$)
Shape: Decreasing on domain and concave up

EXAMPLE: #44 The population in a certain city was 800,000 in 2003, and its future size is predicted to be $P = 800{,}000e^{-0.020t}$ where t is the number of years after 2003.

a. Does this model indicate that the population is increasing or decreasing?

b. Use this model to predict the population of the city in 2010.

c. Use this model to predict the population of the city in 2020.

d. What is the average rate of change in population between 2010 and 2020?

Growth and Decay

For initial amount $y_o > 0$, the equation $y = y_0 b^{kx}$ $b > 1$ defines a growth function if $k > 0$ and a decay function if $k < 0$.

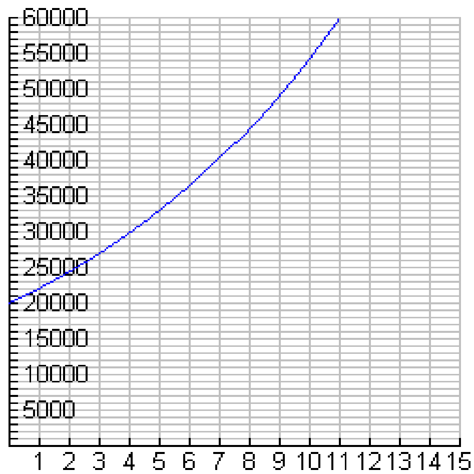
The Number *e*

The number *e* is an irrational number with decimal approximations 2.718281828 (to nine decimal places). It occurs frequently in business and science. The graph is near the graph of above the graph of $y = 2^x$ and below the graph of $y = 3^x$.

Growth of an Investment

If \$20,000 is invested for 12 years at 10%, compounded continuously, the future value is given by $S = 20{,}000e^{0.10t}$ dollars.

a. Graph this function for $0 \leq t \leq 7$.



b. Use this graph to estimate when the future value will be \$50,000.

Homework Course Compass Toolbox chapter 5 and section 5.1 Bookwork page 330 #1, 2, 3, 4, 39