Lesson 4.1 & 4.2(F'11)

Objectives: To transforms graphs:

a) Vertical shifts
b) Horizontal shifts
c) Stretching
d) Compressing
e) Reflecting

To look at the graph to determine if a graph has symmetry.
To find the sum, difference, product and quotient of functions.
To use revenue/cost functions to find profit.
To find the average cost.
To find the composition of functions.

Shifts of Graphs of Functions

Vertical

Let's explore the following graphs on your calculator.

$y = x^2$ $y = x^2 + 4$ $y = x^2 – 2$ $y = x^2 + 6$

Vertical Shifts of Graphs

If k is a positive real number:

The graph of $g(x)= f(x) + k$ can be obtained by shifting the graph of f(x) upward k units.

The graph of $g(x)= f(x) - k$ can be obtained by shifting the graph of f(x) downward k units.

How are these graphs shifted?

$y = \frac{1}{x} + 3$ $y = \frac{1}{x} - 4$ $y = \sqrt{x} - 3$ $y = \sqrt{x} + 4$

Horizontal

Let's explore the following graphs on your calculator.

$y = x^2$ $y = (x - 2)^2$ $y = (x + 3)^2$ $y = (x - 4)^2$

Horizontal Shifts of Graphs

If h is a positive real number:

The graph of g(x) = f(x – h) can be obtained by shifting the graph of f(x) to the right h units.

The graph of g(x) = f(x + h) can be obtained by shifting the graph of f(x) to the left h units.

How are these graphs shifted?

$f(x) = (x - 2)^3$ $f(x) = \sqrt{x-3}$ $f(x) = \frac{1}{x+3}$

Stretching and Compressing Graphs

Let's explore the following graphs on your calculator.

$y = x^2$ $y = 2x^2$ $y = 4x^2$

This is Vertical Stretching.
Let's explore the following graphs on your calculator.

$y = x^2$ $y = \frac{1}{3}x^2$ $y = \frac{1}{4}x^2$

This is Vertical Compressing.

Stretching and Compressing Graphs

The graphs of y = af(x) is obtained by stretching the graph of f(x) by a factor of $|a|$ if $|a| > 1$, and compressing the graph of f(x) by a factor of $|a|$ if $|a| < 1$.

Reflections of Graphs

Let's explore the following graphs on your calculator.

$f(x) = \sqrt{x}$ $f(x) = -\sqrt{x}$ $f(x) = \sqrt{-x}$

Reflections of Graphs Across the Coordinate Axes

1. The graph of y = -f(x) can be obtained by reflecting the graph of y = f(x) across the x-axis.

2. The graph of y = f(-x) can be obtained by reflecting the graph of y = f(x) across the y-axis.

Discuss each of the following graphs.

$f(x) = (-x)^3$ $f(x) = -\sqrt[3]{x}$ $f(x) = -3x^2$ $f(x) = \frac{1}{2}(-x)^2$

Graphing Transformations

In general the graph of a function in the form

$G(x) = af(x - h) + k$ is the graph of $y = f(x)$

shifted h units horizontally, k units vertically and stretched or compressed by a factor of $|a|$. The horizontal shift is to the right when $h > 0$ and to the left when $h < 0$. The vertical shift is up when $k > 0$ and down when $k < 0$. the graph of f is compressed by a factor of $|a|$ when $0 < |a| < 1$and stretched by a factor of $|a|$ when $|a| > 1$. If a is negative, the graph of f is reflected across the x axis. If x is replaced with –x, the graph of f is reflected across the y-axis). (pg 259)

Describe the following graphs.

$y = -2(x - 3)^2 - 5$

$y = -\frac{1}{4}(x - 1)^3 + 2$

$y = 2\sqrt{-x} + 3$

Example # 18(page 266). How is the graph of $y = (x + 4)^3 - 2$ transformed from the graph of $y = x^3$?

Example; Suppose that the graph of $y = x^{1/2}$ is stretched by a factor of 2 and then shifted up 3 units. What is the equation that gives the new graph?

SYMMETRY

Symmetry with Respect to the y-axis

The graph of $y = f(x)$ is symmetric with respect to the y-axis if, for every point (x, y) on the graph the point $(-x, y)$ is also on the graph. That is, $f(-x) = f(x)$ for all x in the domain of f. Such a function is called an **even** function.

Symmetry with Respect to the Origin

The graph of $y = f(x)$ is symmetric with respect to the origin if, for every point (x, y) on the graph the point $(-x, -y)$ is also on the graph. That is, $f(-x) = -f(x)$ for all x in the domain of f. Such a function is called an **odd** function.

Symmetry with Respect to the x-axis

(These are NOT functions)

The graph of $y = f(x)$ is symmetric with respect to the x-axis if, for every point (x, y) on the graph the point $(x, -y)$ is also on the graph.

$x^2 + y^2 = 16$

Operations with Functions

Sum	$(f + g)(x) = f(x) + g(x)$
Difference	$(f - g)(x) = f(x) - g(x)$
Product	$(f \bullet g)(x) = f(x) \bullet g(x)$
Quotient	$(\frac{f}{g})(x) = \frac{f(x)}{g(x)}$ $\quad g(x) \neq 0$

The domain of the sum, difference, and product of *f* and *g* consists of all real numbers of the input variables for which *f* and *g* are defined. The domain of the quotient function consists of all real numbers for which *f* and *g* are defined and $g \neq 0$.

Examples: Given $f(x) = x^2 + 3$ and $g(x) = 3x - 1$, find:

a) $(f + g)(x)$

b) $(f - g)(x)$

c. $(f \bullet g)(x)$

d) $(\frac{f}{g})(x)$ (give the domain)

Given $f(x) = 2x - 4$ and $g(x) = x^2 - 4$, find $(\frac{f}{g})(x)$ and specify the domain.

Revenue, Cost and Profit

If a company sells x units of a product for p dollars per unit, then the total revenue for this product can be modeled by the linear function: $R(x) = px$.

The total cost of producing and selling a product involves two parts, the fixed cost and the variable cost. The fixed cost include such things as rent, utilities, and equipment, and they remain constant. Variable cost are those directly related to the number of units produced. Thus, the cost is found by using the formula: cost = variable costs + fixed cost.

The Profit that a company makes on its product is found by subtracting the total cost of production from the total revenue for the product: $P(x) = R(x) - C(x)$.

Example: #40) A manufacturer of computers has monthly fixed costs of $87,500 and variable costs of $87 per computer, and it sells the computer for $295 per unit.

a) Write the function that models the profit *P* from the production and sale of *x* computers.

b) What is the profit if 700 computers are produced and sold in one month?

c) What is the *y*-intercept of the graph of the profit function? What does it mean?

Average Cost

Average cost:

$\overline{C(x)} = \frac{C(x)}{x}$ C(x) is the total cost and x is the number of items produced.

EXAMPLE: #34The monthly cost of producing x electronic computers is $C(x) = 2.15x + 2350$.

a. Find the monthly average cost function.

b. Find the average cost for the production of 100 components.

Composition of Functions

The **Composite Function,** f of g, is denoted by $(f \circ g)(x)$ and defined by $(f \circ g)(x) = f(g(x))$.

The **Domain** of $(f \circ g)(x)$ is the subset of the domain of g for which $(f \circ g)(x)$ is defined

The composite function $(g \circ f)(x)$ is defined by $(g \circ f)(x) = g(f(x))$.

The domain of $(g \circ f)(x)$ is the subset of the domain of f for which $(g \circ f)(x)$ is defined.

Examples:

1. Given $f(x) = 2x^2 - 5$ and $g(x) = 2x - 6$, find

$(f \circ g)(x)$ and $(g \circ f)(x)$.

2. Given $f(x) = \frac{1}{x+2}$ and $g(x) = 2x + 7$, find

$(f \circ g)(x)$ and $(g \circ f)(x)$.

Determine the domain of each.

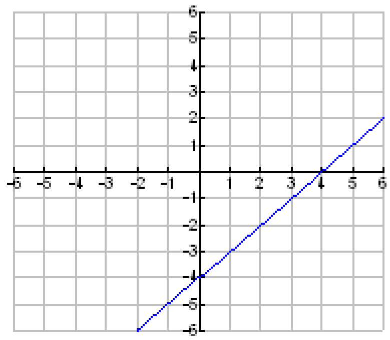
Composite Functions on a Calculator

Example: $f(x) = x^2 - x$ and $g(x) = 4x - 5$.

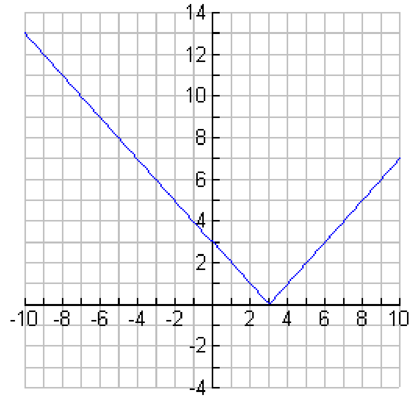
1. Enter Y_1 as $x^2 - x$ and Y_2 as $4x - 5$.
2. Enter $Y_1(Y_2(3))$ to find $f(g(3))$.

   Y_1 and Y_2 can be found by using Vars, Y-vars, Function, Y_1 and Y_2.
3. Find $g(f(3)$.

Example: Use the graphs of f and g below to evaluate the functions.



f



g

a. (f - g)(2)

b) (fg)(-1)

c) $(f \circ g)(3)$

d) $(g \circ f)(-2)$

e. $\left(\frac{f}{g}\right)(5)$

YOU TRY

#39 A manufacturer of satellite systems has monthly fixed costs of $32,000 and variable costs of $432 per system, and it sells the systems for $595 per unit.

a. Write the function that models the profit P from the production and sale of x units of the system.

b. What is the profit of 600 satellite systems are produced and sold in 1 month?

c. At what rate does the profit grow as the number of units increases?

Homework Course Compass 4.1 & 4.2 bookwork pages 266 #27, 31 and pages 278 #24, 26, 35, 38 & Piece Wise Function Supplement