Lesson 4.1 (Fall 2009)

Objectives: To transforms graphs:

a) Vertical shifts
b) Horizontal shifts
c) Stretching
d) Compressing
e) Reflecting

To determine if a graph has the following types of symmetry:

a) y-axis
b) x-axis
c) origin

Shifts of Graphs of Functions

Vertical

Let's explore the following graphs on your calculator.

$y = x^2$ $\quad$ $y = x^2 + 4$ $\quad$ $y = x^2 - 2$ $\quad$ $y = x^2 + 6$

Vertical Shifts of Graphs

If k is a positive real number:

The graph of g(x)= f(x) + k can be obtained by shifting the graph of f(x) upward k units.

The graph of g(x)= f(x) - k can be obtained by shifting the graph of f(x) downward k units.

How are these graphs shifted?

$y = \frac{1}{x} + 3$ $\quad$ $y = \frac{1}{x} - 4$ $\quad$ $y = \sqrt{x} - 3$ $\quad$ $y = \sqrt{x} + 4$

Horizontal

Let's explore the following graphs on your calculator.

$y = x^2$ $y = (x - 2)^2$ $y = (x + 3)^2$ $y = (x - 4)^2$

Horizontal Shifts of Graphs

If h is a positive real number:

The graph of $g(x) = f(x – h)$ can be obtained by shifting the graph of $f(x)$ to the right h units.

The graph of $g(x) = f(x + h)$ can be obtained by shifting the graph of $f(x)$ to the left h units.

How are these graphs shifted?

$f(x) = (x - 2)^3$ $f(x) = \sqrt{x-3}$ $f(x) = \frac{1}{x+3}$

Stretching and Compressing Graphs

Let's explore the following graphs on your calculator.

$y = x^2$ $y = 2x^2$ $y = 4x^2$

This is Vertical Stretching.

Let's explore the following graphs on your calculator.

$y = x^2$ $y = \frac{1}{3}x^2$ $y = \frac{1}{4}x^2$

This is Vertical Compressing.

Stretching and Compressing Graphs

The graphs of y = af(x) is obtained by stretching the graph of f(x) by a factor of $|a|$ if $|a| > 1$, and compressing the graph of f(x) by a factor of $|a|$ if $|a| < 1$.

Reflections of Graphs

Let's explore the following graphs on your calculator.

$f(x) = \sqrt{x}$ $f(x) = -\sqrt{x}$ $f(x) = \sqrt{-x}$

Reflections of Graphs Across the Coordinate Axes

1. The graph of y = -f(x) can be obtained by reflecting the graph of y = f(x) across the x-axis.

2. The graph of y = f(-x) can be obtained by reflecting the graph of y = f(x) across the y-axis.

Discuss each of the following graphs.

$f(x) = (-x)^3$ $f(x) = -\sqrt[3]{x}$ $f(x) = -3x^2$ $f(x) = \frac{1}{2}(-x)^2$

Graphing Transformations

In general the graph of a function in the form

$$G(x) = af(x - h) + k$$ is the graph of $y = f(x)$

shifted h units horizontally, k units vertically and stretched or compressed by a factor of $|a|$. The horizontal shift is to the right when $h > 0$ and to the left when $h < 0$. The vertical shift is up when $k > 0$ and down when $k < 0$. the graph of f is compressed by a factor of $|a|$ when $0 < |a| < 1$and stretched by a factor of $|a|$ when $|a| > 1$. If a is negative, the graph of f is reflected across the x axis. If x is replaced with –x, the graph of f is reflected across the y-axis). (pg 259)

Describe the following graphs.

$y = -2(x - 3)^2 - 5$

$y = -\frac{1}{4}(x - 1)^3 + 2$

$y = 2\sqrt{-x} + 3$

Example # 18(page 266). How is the graph of $y = (x + 4)^3 - 2$ transformed from the graph of $y = x^3$?

Example; Suppose that the graph of $y = x^{1/2}$ is stretched by a factor of 2 and then shifted up 3 units. What is the equation that gives the new graph?

#46 The percent of persons 12 years of age and over in the U.S. who said that they had used marijuana at least once within the month prior to being asked during selected years is described by the function $M(x) = -\frac{1}{12}\left(x - \frac{789}{10}\right)^2 + \frac{15,541}{1200}$ when $79 \le x \le 88$, where x is the number of years after 1900.

a. The graph of this function is a shifted graph of which basic function?

b. Find and interpret M(79).

c. Sketch a graph of M.

SYMMETRY

Symmetry with Respect to the y-axis

The graph of $y = f(x)$ is symmetric with respect to the y-axis if, for every point (x, y) on the graph the point $(-x, y)$ is also on the graph. That is, $f(-x) = f(x)$ for all x in the domain of f. Such a function is called an **even** function.

Examples: a) Show that the following function is even.

$$y = x^2 - 2$$

Symmetry with Respect to the Origin

The graph of $y = f(x)$ is symmetric with respect to the origin if, for every point (x, y) on the graph the point $(-x, -y)$ is also on the graph. That is, $f(-x) = -f(x)$ for all x in the domain of f. Such a function is called an **odd** function.

Example: Show that the following function is odd.

$$y = x^3 - 4x$$

Symmetry with Respect to the x-axis

(These are NOT functions)

The graph of $y = f(x)$ is symmetric with respect to the x-axis if, for every point (x, y) on the graph the point $(x, -y)$ is also on the graph.

Determine **algebraically** if the following are symmetric with respect to the x-axis, the y-axis, or the origin.

	$y = x^3 - x$	$x^2 + y^2 = 16$
Even y-axis (x, y) (-x, y)		
Odd-origin (x, y) (-x, -y)		
x-axis (x, y) (x, -y)		

Which of the above equations are even, odd, or neither?

Homework Course Compass 4.1 bookwork pages 266 #27, 31,

DUE BY________________________