Lesson 1.3 & 1.4(F'09)

Objectives: 1. To determine if an equation is linear or nonlinear
2. To find the slope of a linear function, and the rate of change
3. To find the intercepts of a linear function
4. To determine if two linear functions are parallel, perpendicular or neither
5. To write the equation of a line in:
   a) general form
   b) slope intercept form
6. To find the average rate of change
7. To solve application problems involving linear equations

LINEAR FUNCTION

A function f defined by f(x) = ax +b where a and b are constants is called a **linear function**.

y = ax + b is also a linear function.

Note x and y must be in separate terms and must be in the numerator with a power of one .

Determine if these are linear functions.
Give the domain and range of all linear functions:

1) 3x +2y = 8 2) x + y = 3 3) y = 9 4) x = 5

5) y = x 6) $y = \frac{1}{x}$ 7. $y = x^3$ 8) xy = 9

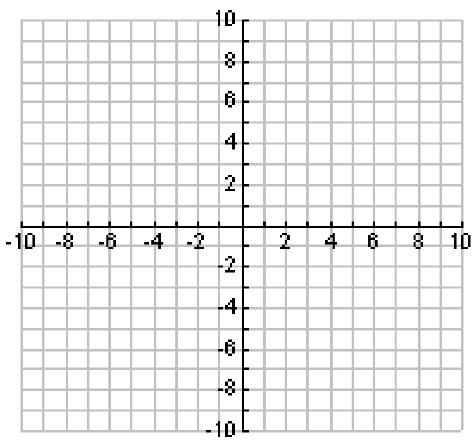
#3 above is a special linear function called a **constant function**.
[y = f(x)= b]

#5 above is called the **identity function** [y = f(x) = x]

Intercepts: The points where a graph intersects the x-axis and y-axis are called the intercepts.
These points are <u>necessary</u> for a complete graph.

<u>Finding the Intercepts Algebraically</u>:
* To find the y-intercepts of the graph of $y = f(x)$, set $x = 0$ and solve for y.
* To find the x-intercepts of the graph of $y = f(x)$, set $y = 0$ and solve for x.

Example: Find the intercepts of this function algebraically, then use these points to sketch the graph of : $4x + 3y = 24$.



Note: If we know the intercepts of a line that we want to graph on our calculators, <u>it will help us set the window values for the complete graph</u>.

To find the x and y intercepts with our calculator- First graph the function in a window that shows all intercepts.
To find the y-intercept, use <u>**TRACE**</u> until x = 0 and y intercept will be displayed.
To find the x-intercept of the graph, use the <u>**ZERO**</u> key under the <u>**CALC**</u> menu

Slope of a Line

The slope of a line is a numerical measure of the steepness of the line. (Tells how the line slants.)

Formula for Slope

Given any two different points on a line, (x_1, y_1) and (x_2, y_2),

the slope, is: $\frac{y_2 - y_1}{x_2 - x_1}$ or $\frac{\textit{vertical change}}{\textit{horizontal change}}$ or $\frac{\Delta y}{\Delta x}$.

Examples: Find the **slopes** of the lines containing these pairs of points.

a) (2, 5) and (1, 1) b) (3, 7) and (6, 7)

Now let's graph a linear function on our calculators.

Graph y = 3x + 5, using Zoom 6.
Look at the TABLE(2[nd] graph) for this function.
What happens to y each time x increases by 1?

What is the slope of this line?____________

Notice the relationship between the slope and the equation of the line.

Can you find the y-intercept from the table?

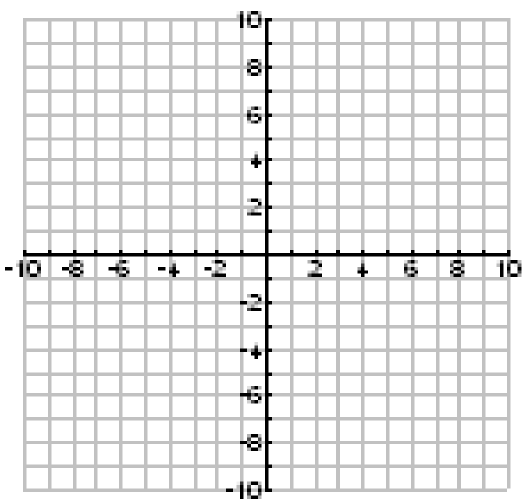
Notice the relationship between the y-intercept and the equation of the line.

Slope-Intercept Form for the Equation of a Line

y = mx + b is the equation of the line with slope m and y-intercept b.

Positive-increasing-line rises as we move from left to right
Negative-decreasing-line falls as we move from left to right
Undefined-"No slope" –vertical line

Draw a line with each of the above slopes (+, -, NO SLOPE)



Interpreting slope

Example: Determine the slope and y intercept of the equation
y = 0.2x – 15.

Determine if the graph is rising or falling.

Graph this equation on the window [-20, 85] (find and appropriate y-range)

Evaluating Linear Models

Linear functions often model real world applications.
Let's look at some.

Example#36 page 57 The percent of the population voting in presidential elections is given by $V(t) = 63.20 - 0.26t$, where t is the number of years after 1950.

a. Why is this a linear function?

b. What is the slope? What does this tell you about the percent of the population voting in the presidential election since 1950?.

c. If this rate continues when will no one vote?

EXAMPLE: #41 (page 58) For the years 1991-2006, the percent *p* of high school seniors who have tried cigarettes can be modeled by $p = 75.751 – 0.743t$, where t is the number of years after 1991.

a. Is the rate of change of the percent positive or negative?

b. How fist is the percent of seniors who tried cigarettes during this period changing. Use the units in the problem in answering your question.

The rate of change of the linear function $y = mx + b$ is the constant m, the slope of the graph of the function.

The profit that a company makes on a product is the difference between the amount it sells its product for (revenue) and its cost.

We will represent the revenue from the sale of *x* units by R(x) and represent the cost for the production and sale of *x* units by C(x). The profit from the production and sale of x units is given by the function:

$$P(x) = R(x) - C(x)$$

For total cost, total revenue, and profit functions that are linear, their rates of change are called **marginal cost**, **marginal revenue**, and **marginal profit**, respectively.

EXAMPLE: #59 (page 59) Suppose the monthly total revenue for the manufacture of golf balls is $R(x) = 1.60x$, where x is the number of balls sold each month.

a. What is the slope of the graph of the total revenue function?

b. What is the marginal revenue for the product?

c. Interpret the marginal revenue for this product.

Writing Equations of Lines

The equation y = mx + b is the equation of a line with slope m and y intercept (0, b).

In applied context, m is the rate of change and b is the initial value (when x = 0)

EXAMPLES:

1) Write the equation of a line with slope 6 and y intercept (0, -4).

2) A appliance repair store charges $50 for a service call and $75 per hour for each hour spent on the repair. If a linear function models the service call charges, write the equation of the function.

3) Write the equation of a line connecting the points (4, 7) and (-3, -8).

a) Method 1 (slope formula)

b) Method 2 (point slope form)

Point-Slope Form of the Equation of a Line

The equation of the line with slope m that passes through a known point (x_1, y_1) is :

$$y - y_1 = m(x - x_1).$$

Use either method to write the equation of a line with a slope of $\frac{1}{2}$ and containing the point (-2, 7).

In general: the equation of a horizontal line with y-intercept b is: **y = b and** the equation of a vertical line with x intercept a is: **x = a**.

Is a horizontal line a function? Vertical line?

What is the slope of a horizontal line?   Vertical line?

Another form of a linear equation is the general form.

General Form of The Equation of a Line

ax + by = c where a, b, and c are real numbers with a and b not both equal to zero.

EXAMPLE: Write the equation $y = \frac{1}{4}x - 4$ in general form.

Forms of Linear Equations

General Form	$ax + by = c$	a, b, c are real number , with a and b not both equal to 0
Point-slope form	$y - y_1 = m(x - x_1)$	Where m is the slope of the line and (x_1, y_1) is a point on the line
Slope-intercept form	$y = mx + b$	where m is the slope and b is the y intercept
Vertical line	$x = a$	Where a is a constant, and a is the x-coordinate of any point on the line.
Horizontal Line	$y = b$	Where b is a constant, and b is the y-coordinate of any point on the line.

EXAMPLE: #39 A business uses straight-line depreciation to determine the value of y of an automobile over a 5-year period. Suppose the original value (when t = 0) is equal to $26,000 and the salvage value (when t = 5) is equal to $1000.

a. By how much has the automobile depreciated over the five years?

b. By how much is the value of the automobile reduced at the end of each of the 5 years?

c. Write the equation that models the value s of this automobile at the end of the year t.

EXAMPLE #42 The monthly premium for a $100,000 life insurance policy for males aged 27-32 are shown in the table below. Write the linear model for the data in the table.

Age (years)	27	28	29	30	31	32
Premium (dollars per month)	11.81	11.81	11.81	11.81	11.81	11.81

EXAMPLE: #56 page 75 The percent of eligible people voting in presidential elections can be expressed as a linear function of the years 1960-2004. The percent was 63.1 in 1960 and 55.3 in 2004.

a. What is the slope of the line joining the given points?

b. What is the average rate of change in the percent voting in these elections? Interpret this value.

c. Use the slope from part (a) and the percent of people voting in 1960 election to write the equation of the line.

EXAMPLE: The body mass index BMI is used to assess the level of fat in a person's body. When Tim weighed 147 pounds his BMI was 23.4. When his weight went to 185, his BMI was 29.5.
Write the equation that models this information.

Parallel and Perpendicular Lines

Two different lines are parallel if their slopes are equal.
Two lines are perpendicular if their slopes are negative reciprocals $m_2 = -\frac{1}{m_1}$ or $m_1 m_2 = -1$.

Are the following lines parallel, perpendicular or neither?

1. $y = 3x - 7$ and $y = -3x + 9$

2. $2x + y = 2$ and $y = \frac{1}{2}x + 8$

EXAMOLE: Write the equation of a line passing through (-3, 7) and perpendicular to the line with equation $2x + 3y = 7$.

Average Rate of Change

The average rate of change of f(x) with respect to x over the interval from x = a to x = b (where a < b) is calculated as

Average rate of change = $\frac{\textit{change in } f(x) \textit{ values}}{\textit{corresponding change in x values}} = \frac{f(b) - f(a)}{b - a}$

EXAMPLE: For the function $y = x^2 + 4$, compute the average rate of change between $x = -2$ and $x = 3$.

EXAMPLE: (#50) (page 74)

The following table gives the number of drinks and the resulting blood alcohol percent for a 180 lb man legally considered driving under the influence (DUI).

Number of drinks	5	6	7	8	9	10
Blood Alcohol Percent	0.11	0.13	0.15	0.17	0.19	0.21

a. The average rate of change of the blood alcohol percent with respect to the number of drinks is a constant. What is it?

b. Use the rate of change and one point determined by a number of drinks and the resulting blood alcohol percent to write the equation of a linear model for this data.

EXAMPLE #30 page 71 For the function f(x) = $3x^2+1$

Find $\frac{f(x+h)-f(x)}{h}$

Homework: Course Compass for section 1.3 & 1.4 and bookwork

page 55 #13, 14, 17, 29

page 70 #27, 51, 58, 61

DUE by ______________________________