

Lesson 11.1 & 11.2

Summer 2008

- Objectives: To solve quadratic equations by:
1. Square Root Property
 2. Completing the square.
 3. Using the Quadratic Formula
 4. To use the discriminant to indicate the type of solutions for the quadratic equation.
 5. To find the x and y intercepts of a quadratic function.
 6. To solve application problems.

Quadratic equation-a second degree equation in one variable, containing a variable with an exponent of 2.

$$ax^2 + bx + c = 0\text{-standard form.}$$

Example: Solve:

1. $x^2 = 81$

There is another way to solve this equation. Notice that the solutions are numbers that can be squared to equal 81. These numbers are the square roots of 81, which are 9 and -9.

Let's state this as a rule.

The Square Root Principle

For any real number a , $x^2 = a$ if and only if $x = \sqrt{a}$ or $-\sqrt{a}$.

Solve:

Now let's use this property to solve equations in the form
 $x^2 = a$.

Solve:

1. $x^2 = 121$

2. $x^2 = 45$

3. $x^2 + 9 = 0$

4. $5n^2 = 12$

5. $2x^2 + 11 = 65$

6. $(x - 4)^2 = 18$

7. $2(x - 3)^2 - 8 = 22$

SOLVE BY COMPLETING THE SQUARE.

1. $x^2 + 8x + 15 = 0$

Solving a quadratic by Completing the Square

To solve a quadratic equation by completing the square:

1. Write the equation in the form $x^2 + bx = c$
2. Complete the square by adding $(b/2)^2$ to both sides.
3. Write the completed square in factored form.
4. Use the square root principle to eliminate the square.
5. Isolate the variable.
6. Simplify as needed.

Solve each of the following by completing the square:

1. $x^2 - 12x - 5 = 0$

2. $9x^2 + 18x = -1$

Instead of completing the square on each problem, let's complete the square on the general quadratic equation and derive a formula.

Let's derive the Quadratic Formula

$$ax^2 + bx + c = 0$$

Quadratic Formula

$$ax^2 + bx + c = 0 \text{ if and only if } x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}.$$

Now let's use it to solve:

1. $x^2 + 3x - 8 = 0$

$a = \underline{\hspace{2cm}}$ $b = \underline{\hspace{2cm}}$ $c = \underline{\hspace{2cm}}$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-3 \pm \sqrt{(3)^2 - 4(1)(-8)}}{2(1)} = \frac{-3 \pm \sqrt{41}}{2}$$

$$2. x^2 - 9x = 41$$

First you must write your equation in the correct form.

$$\underline{\hspace{10em}} = 0$$

$$a = \underline{\hspace{2em}} \quad b = \underline{\hspace{2em}} \quad c = \underline{\hspace{2em}}$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} =$$

$$3. 3x^2 + 10x = 8$$

$$a = \underline{\hspace{2em}} \quad b = \underline{\hspace{2em}} \quad c = \underline{\hspace{2em}}$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Choosing a method for Solving Quadratic Equations

We have now studied several methods for solving a quadratic equation. The following table summarizes the methods.

Method	When the Method is Best
1. Factoring	If the quad. can be factored easily
2. Square Root	If quad can easily be written in $ax^2 = c$ or $(ax + b)^2 = c$
3. Completing the square	Rarely used
4. Quadratic formula	When factoring is not easy.

The expression $b^2 - 4ac$ is called the **discriminant**.

Given a quadratic equation in the form $ax^2 + bx + c = 0$, where $a \neq 0$, to determine the number and type of solution the quadratic has we will evaluate the discriminant. If

$b^2 - 4ac < 0$, then the equation has **two non real** (complex) solutions

$b^2 - 4ac = 0$, then the equation has **one real** solution

$b^2 - 4ac > 0$, then the equation has **two real** solutions.

Find the value of the discriminant, and use it to determine the number and type of solutions.

1. $x^2 + 7x - 3 = 0$

$b^2 - 4ac$

$$2. x^2 - 8x + 16 = 0$$

$$b^2 - 4ac$$

$$3. x^2 + 6x + 18 = 0$$

$$b^2 - 4ac$$

Graph of $f(x) = ax^2 + bx + c$

When we graph a quadratic function three “nice” points are the x and y intercepts.

Remember to find the x intercept we let $y = 0$, and to find the y intercept we let $x = 0$.

Graph $y = x^2 - 2x - 8$ by first finding the x and y intercepts.

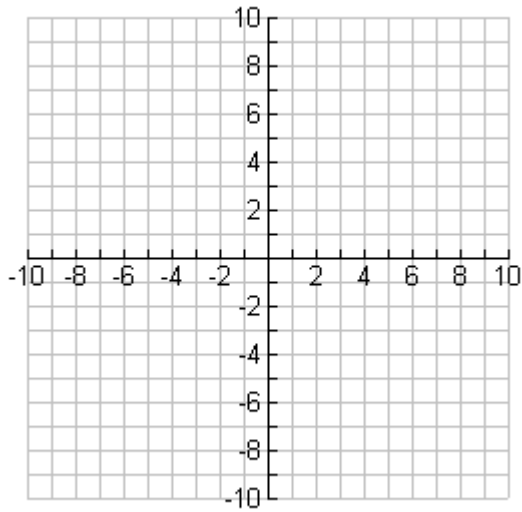
$$1. \text{ Let } y = 0 \quad 0 = x^2 - 2x - 8 \quad 0 = (x - 4)(x + 2)$$

The x intercepts are _____ and _____

$$2. \text{ Let } x = 0 \quad y = (0)^2 - 2(0) - 8 =$$

The y intercept is _____.

Now let's make a table and graph this function.



x	x^2	x^3
-4	16	-64
-3	9	-27
-2	4	-8
-1	1	-1
0	0	0
1	1	1
2	4	8
3	9	27
4	16	64
5	25	125
6	36	216
7	49	343
8	64	512
9	81	729

Problem Solving

Translate to a quadratic equation, then solve.

1. The length of a rectangular floor is 2 meters less than three times its width. If the area is 96 meters, find the length and width.

2. On a 135 mile bicycle excursion, Maria averaged 5 mph faster for the first 60 miles than she did for the last 75 miles. The entire trip took 8 hours. Find her rate for the first 60 miles.

3. Using a riding mower, Fran can mow the grass in 4 hours less time than it takes Donnie using a push mower. Together they can mow the grass in $2\frac{2}{3}$ hours. How long does it take each working alone?

Worker	Time alone	Rate of work	Time at work	Portion of job com
Fran	$x - 4$	$\frac{1}{x-4}$	$\frac{8}{3}$	$\frac{8}{3(x-4)}$
Donnie	x	$\frac{1}{x}$	$\frac{8}{3}$	$\frac{8}{3x}$

CW Worksheet 11.1& 11.2

Homework Course Compass 11.1 & 11.2 and

Final review # _____ - # _____