Lesson 3.1, 3.2 (Fall 2009)

Objectives:
1. To graph quadratic functions (parabolas) by finding :
   a) vertex (max or min)
   b) intercepts
   c) axis of symmetry
   d) intervals of increase and decrease
2. To solve quadratic equations.
3. To solve application problems.

Parabola- is the graph of a quadratic function. A parabola has a turning point called the **vertex**.

General Form of a Quadratic Function-

f(x) = $ax^2 + bx + c$ for $a \neq 0$. Vertex $\left(\frac{-b}{2a}, f\left(\frac{-b}{2a}\right)\right)$.

The **domain** of a quadratic function is all real numbers.

The graph of a quadratic function opens up if a is positive, and is **concave up**. The vertex point is a **minimum**.

The graph of a quadratic function opens down if *a* is negative, and is **concave down**. The vertex is a **maximum**.

The vertical line through the vertex is called the **axis of symmetry** because it divides the graph into two halves that are reflections of each other.

Increasing and Decreasing Functions

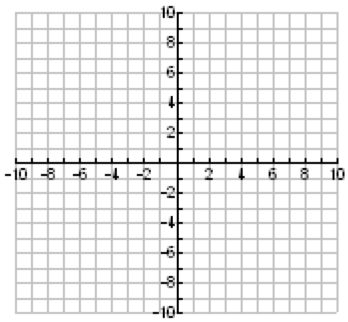
A function is increasing on an interval if, for in the interval, when $x_2 > x_1$ it is true that $f(x_2) > f(x_1)$.

A function is decreasing on an interval if, for any x_1 and x_2 in the interval, when $x_2 > x_1$ it is true that $f(x_2) < f(x_1)$.

Now let's look at two graphs.

EXAMPLE. Graph each parabola and fill in the blanks below.

1. $y = x^2 + 1$



Vertex_______________

Axis of symmetry________

Opens____________

Increases ____________

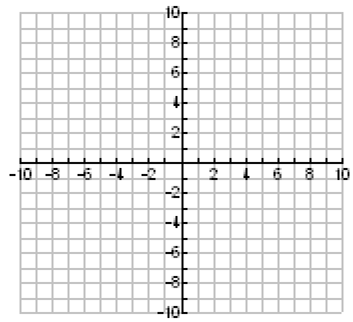
Decreases____________

Concave_______

Domain__________

Range________

2. $y = -x^2 + 3x - 4$



Vertex_______________

Axis of symmetry________

Opens____________

Increases ____________

Decreases____________

Concave_______

Domain__________

Range________

Role of a in the general equation.

a = 1 "normal

$|a| > 1$ rises rapidly "thin"

$|a| < 1$ rises slowly "fat"

see summary page 176

Standard Form of a Quadratic Equation

$y = a(x - h)^2 + k$ is a quadratic function with vertex at (h, k) and axis of symmetry x = h. (Note the role of *a* is the same)

Example: Find the vertex and the equation of the axis of symmetry for each parabola. Is the vertex a max. or a min.?

1. $f(x) = 2(x - 5)^2 + 3$

Vertex______________

Axis of symmetry_______

Max or Min__________

2. $f(x) = -\frac{2}{3}(x + 6)^2 - 2$

Vertex______________

Axis of symmetry_______

Ma x or Min_________

Example: Find the equation of a parabola with the following points.

x	-1	0	1	2	3
y	13	-2	-7	-2	13

Remember to show a complete graph you must show: vertex, axis of symmetry, and the intercepts

Example: Graph $y = -0.2x^2 - 4x + 18$ on your calculator. Find the x coordinate of the vertex, set your viewing window so that the vertex is near the center of the window.

Example: Suppose that the manufacturer of a gas clothes dryer has found that, when the unit price is p dollars, the revenue R (in dollars) is: $R(p) = -4p^2 + 4000p$.
What unit price for the dryer should be established to maximize revenue? What is the maximum revenue?

Example: If 400 feet of fence are sued to enclose a rectangular field, the resulting area of the field is
$A = x(200 - x)$, where x is the width of the pen. What is the maximum area of the pen?

Example: The profit for a product is given by
$P = 1600 - 100x + x^2$, where x is the number of units produced and sold. Graphically, find the x-intercepts of the graph of this function to find how many units will give break even (that is, return a profit of zero)

Solving Quadratic Equations

Quadratic Equation in One Variable- $ax^2 + bx + c = 0$ when a, b, c are real numbers and $a \neq 0$.

Factoring—Zero product Property
For real numbers a and b, the product ab = 0 iff either a = 0 or b = 0.

Example: Solve by factoring. Check by graphing.

1. $3x^2 - 12x = 0$

2. $6x^2 - 13x + 6 = 0$

Example: Solve by graphing.

The solutions to $ax^2 + bx + c = 0$ is(are) the x intercept(s) of $y = ax^2 + bx + c$

1. $0.1x^2 - 0.2x = 0.5$

Example: Solve using the

Square Root Property- If $x^2 = a$ then $x = \pm\sqrt{a}$ if $\sqrt{a}$ is real.

1. $x^2 = 45$

2. $25x^2 = 49$

Example: Solve using the Quadratic formula

If $ax^2 + bx + c = 0$, then $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$.

1. $2x^2 = x + 4$

2. $6x^2 = -3x - 2$

Discriminant $b^2 - 4ac$ if

a) $b^2 - 4ac = 0$________________

b) $b^2 - 4ac > 0$ ________________

c) $b^2 - 4ac < 0$ ________________

Example #52 The profit for a product is given by $P(x) = -15x^2 + 180x - 405$ thousand dollars, where x is the number of tons of product produced and sold. How may tons give break-even (zero profit) for this product?

Example: #54 The total revenue function for a product is given by $R = 266x$, and the total cost function for this same product is $C = 2000 + 46x + 2x^2$, where *R* and *C* are each measured in thousands of dollars and *x* is the number of units produced and sold.

a. Form the profit function for this product from the two given functions.

b. What is the profit when 55 units are produced and sold?

c. How many units must be sold to break even on this product?

Example. #60The demand for a product is given by $p = 7000 - 2x$ dollars, and the supply for this product is given by $p = 0.01x^2 + 2x + 1000$ dollars, where *x* is the number of units demanded and supplied when the price per unit is *p* dollars. Find the equilibrium quantity and equilibrium price.

Homework Course Compass Toolbox 2 and sections 3.1and 3.2 and bookwork page 182 # 12, 39, 49 and page 199 # 6, 35, 47, 55, and Supplement #1