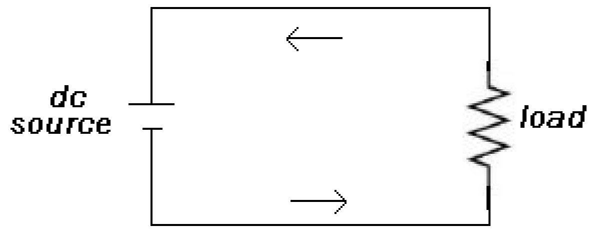
Unit 22 AC Circuits, RLC Circuits, Power in an AC Circuit, & Transformers

Alternating Current versus Direct Current

The figure below shows the schematic diagram of a very basic DC circuit.

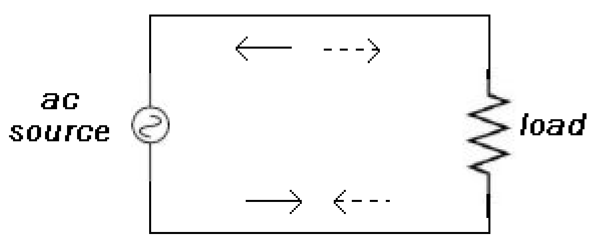


It consists of nothing more than a source (a producer of electrical energy) and a load (whatever is to be powered by that electrical energy). The source can be any electrical source: a chemical battery, an electronic power supply, a mechanical generator, or any other possible continuous source of electrical energy. The source in this figure is a battery.

At the same time, the load can be any electrical load: a light bulb, electronic clock or watch, electronic instrument, or anything else that must be driven by a continuous source of electricity. The figure here represents the load as a simple resistor.

Regardless of the specific source and load in this circuit, electrons leave the negative terminal of the source, travel through the circuit in the direction shown by the arrows, and eventually return to the positive terminal of the source. This action continues for as long as a complete electrical circuit exists.

Now consider the same circuit with a single change, as shown in the next figure.



This time, the energy source is constantly changing. It begins by building up a voltage which is positive on top and negative on the bottom, and therefore pushes electrons through the circuit in the direction shown by the solid arrows. However, then the source voltage starts to fall off, and eventually reverses polarity. Now current will still flow through the circuit, but this time in the direction shown by the dotted arrows. This cycle repeats itself endlessly, and as a result the current through the circuit reverses direction repeatedly. This is known as an ***alternating current.***

This kind of reversal makes no difference to some kinds of loads. For example, the light bulbs in your home don't care which way current flows through them. When you close the circuit by turning on the light switch, the light turns on without regard for the direction of current flow.

Of course, there are some kinds of loads that require current to flow in only one direction, such as the power supplies used in some of the labs. In such cases, alternating circuit from a wall socket is converted to direct current.

Resistors in an AC Circuit and rms Current & Voltage

Again, direct current is electric current whose flow of charge is always in one direction. Alternating current, ac, such as that produced by a 60 hertz generator, rapidly changes directions. In a previous unit, we saw that the output of an ac generator is sinusoidal and varies with time according to

$$V(t) = V_{max} sin 2\pi f t, \text{ where } V_{max} = NBA\omega.$$

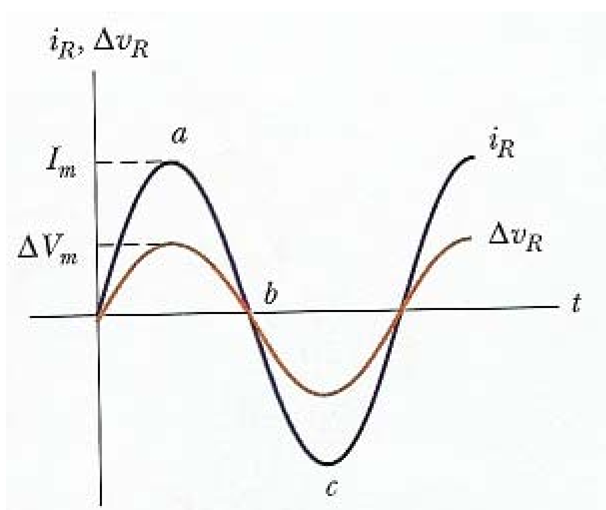
The potential V oscillates between V_{max} and $-V_{max}$. V_{max} is referred to as the peak voltage. The frequency f is the number of oscillations made per second. In most areas of the U.S. and Canada, f is 60 Hz. In some countries, 50 Hz is used.

The current also varies with time as

$$I(t) = I_{max} sin 2\pi f t$$

The current is considered positive when the current flows in one direction and negative when it flows in the other direction.

The current and the voltage across the resistor are shown in the figure below.



At point a on the curve, the current has a maximum value in one direction. Between points a and b, the current is decreasing in magnitude but is still in the positive direction. At point b, the current is momentarily zero. The current then begins to increase in the opposite direction between points b and c. At point c, the current has reached its maximum value in the negative direction.

It's important to note that the current and voltage are in step with each other because they vary indentically with time. Because the current and voltage reach their maximum and minimum values at the same time, they are said to be in phase.

Using Power to Measure Alternating Current

Alternating current is as often positive as it is negative. Thus, the average current is zero. This does not mean, however, that no power is needed or that no heat is produced in a resistor. Electrons do flow back and forth, and do produce heat.

It's important to consider the power of a resistor, because it is difficult to use a meter to measure alternating current, since it is not constant like direct current. So the meter is designed to measure what is called the ***effective,or the rms current***, which is the direct current that generates heat in a resistor R at the same rate as alternating current. Therefore, the average power generated in an ac circuit is equal to the power generated in a dc circuit carrying an effective current i_{eff} or i_{rms}. Mathematically, we can write

$$(P)_{dc} = (P_{avg})_{ac}$$

Recall that $P = i^2 R$. So the above equation becomes

$$i_{rms}^2 R = \frac{i_{max}^2 R}{2}$$

By dividing both sides by R and then taking the square root, we get.

$$i_{rms} = \frac{i_{max}}{\sqrt{2}}$$

rms stands for root mean square.

An ac circuit also has an effective-rms voltage given by

$$V_{rms} = \frac{V_{max}}{\sqrt{2}}$$

The effective voltage is a constant value of the voltage that produces the same effect as the ac voltage.

The rms voltage across a resistor is equal to the rms current in the circuit times the resistance.

$$V_{rms} = i_{rms}R$$

Problem

An ac voltage source has an output of $V = 200\text{V} sin 2\pi f t$. This source is connected to a 100 Ω resistor. Find the rms voltage and current, as well as the peak current.

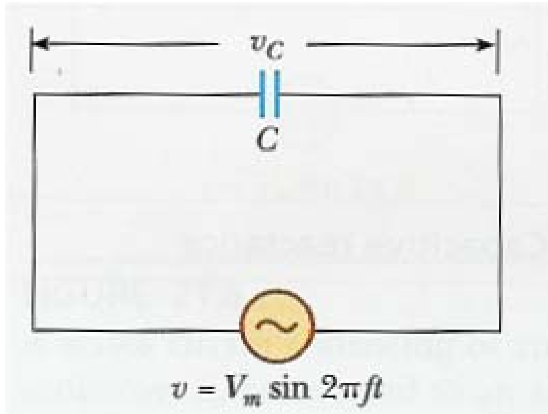
Solution

$V_{rms} = \frac{V_{max}}{\sqrt{2}} = \frac{200\text{ V}}{\sqrt{2}} = 141\text{ V}$

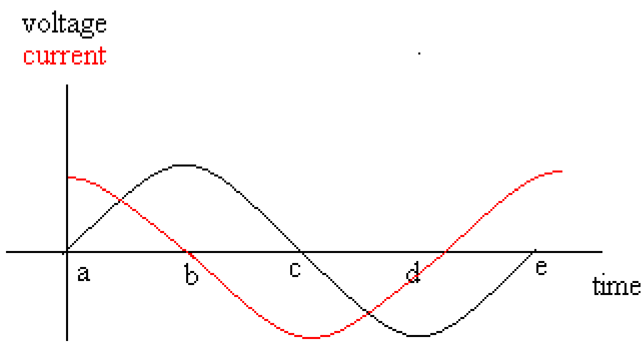
$i_{rms} = \frac{V_{rms}}{R} = \frac{141\text{ V}}{100\ \Omega} = 1.41\text{ A}$

$i_{max} = \sqrt{2}\, i_{rms} = 1.41\sqrt{2} = 2\text{A}.$

Capacitors In An ac Circuit



If an ac circuit consists of a generator and a capacitor, the voltage lags behind the current by 90°. That is, the voltage reaches its maximum value one quarter of a period after the current reaches its maximum value.



To see this, consider that a capacitor charges until the voltage across its plates matches that of the emf. In this case, the emf is an ac generator. Since the charge on the plates is proportional to the voltage, when the voltage is zero, the charge is zero; and, when the voltage is maximum the charge is maximum. But what about the current i? At point a when the voltage starts increasing, the charge on the plates is zero. Thus charge flows freely toward the plates and the current is large. As the voltage approaches its maximum value, the charge that accumulated on the plates tend to prevent more charge from flowing, so the current drops to zero at point b. The accumulated charge now starts to flow off the plates and the magnitude of the current again increases, but in the opposite direction, it reaches a maxium when the voltage is at point c. The behavior depicted in the above graph repeats itself over and over.

The impeding effect of a capacitor on current in an ac circuit is called ***capacitive reactance***, X_c, defined by

$$X_c = \frac{1}{2\pi f C}$$

Problem

A 1.0 µF capacitor is connected in series with a 60 Hz ac voltage source. If the $V_{rms} = 120$ V, find the peak and rms currents.

Solution

First, find the peak current, $i_{max} = \frac{V_{max}}{X_C}$.

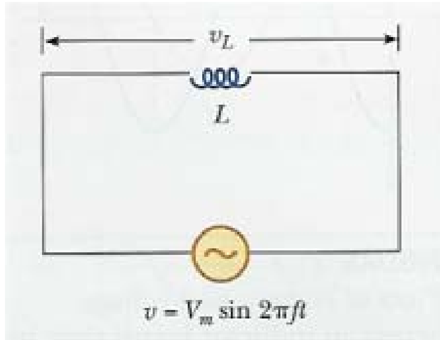
$V_{max} = \sqrt{2} V_{rms} = \sqrt{2} \cdot 120\,\text{V} = 170\,\text{V};$ and

$X_C = \frac{1}{2\pi f C} = \frac{1}{2\pi(60s^{-1})(1.0\times10^{-6}\text{F})} = 2.7 \times 10^3\,\Omega.$

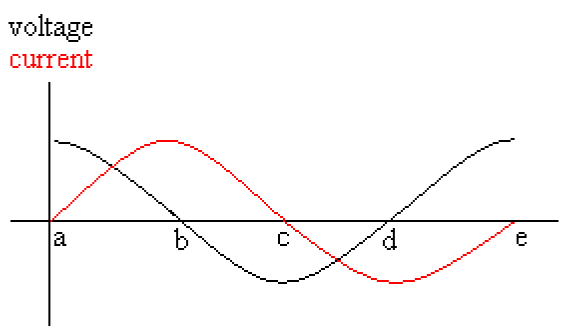
So $i_{max} = \frac{V_{max}}{X_C} = \frac{170\,\text{V}}{2.7\times10^3\,\Omega} = 63$ mA

Next, $i_{rms} = \frac{V_{rms}}{X_C} = \frac{120\,\text{V}}{2.7\times10^3\,\Omega} = 44$ mA

Inductors In An ac Circuit



If an ac circuit consists of a generator and an inductor, the voltage leads the current by 90°. That is, the voltage reaches its maximum value one quarter of a period before the current reaches its maximum value.



Inductors do not behave the same as resistors. Whereas resistors simply oppose the flow of electrons through them (by dropping a voltage directly proportional to the current), inductors oppose changes in current through them, by dropping a voltage directly proportional to the rate of change of current. In accordance with Lenz's Law, this induced voltage is always of such a polarity as to try to maintain current at its present value. That is, if current is increasing in magnitude, the induced voltage will "push against" the electron flow; if current is decreasing, the polarity will reverse and "push with" the electron flow to oppose the decrease. This effective resistance is called ***inductive reactance***, X_L, where

$$X_L = 2\pi f L$$

where f is the frequency, and L is the inductance. *(Inductance is the property of a coil that causes it to resist the flow of current. Inductance is measured in Henries)*

Problem

A coil, with an inductance of 0.300 H, is placed in series with aa 60 Hz ac, 120 V (rms) voltage source. Determine the rms current.

Solution

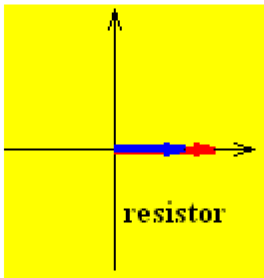
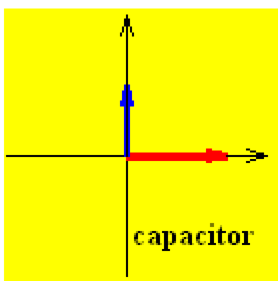
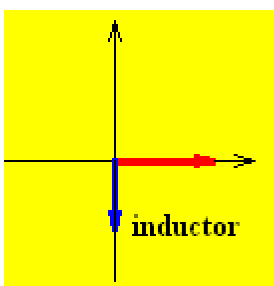
$i_{rms} = \frac{V_{rms}}{X_L}$

$X_L = 2\pi f L = (6.28)(60.0\, s^{-1})(0.300\,\text{H}) = 113\,\Omega$

Therefore, $i_{rms} = \frac{120\text{ V}}{113\,\Omega} = 1.06\text{ A}$

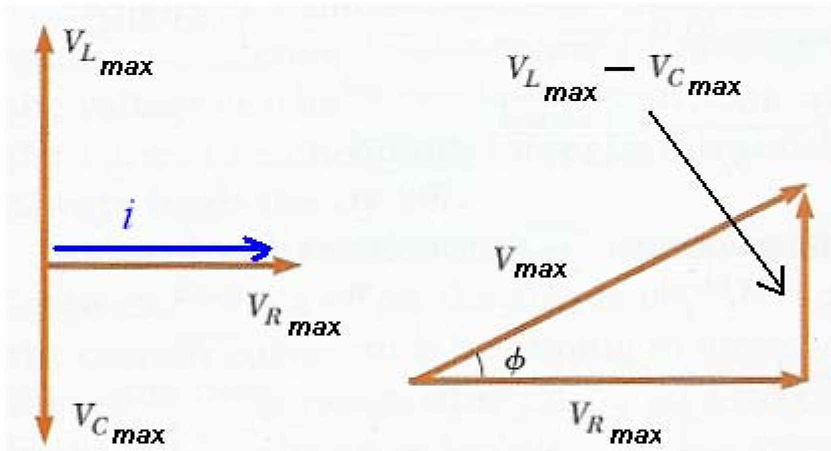
RLC Circuits

In the above discussion, we examined individually the effects that a resistor, a capacitor, and an inductor have when each is placed in an ac circuit. Each, in its own way, resists the flow of current in the circuit. So they should collectively resist the flow of current; and, they do. This combined resistance is called ***impedence.***

	The instantaneous voltage across resistor is in phase with the current.
	The instantaneous voltage of a capacitor lags the current by 90 degrees.
	The instantaneousl voltage of an inductor leads the current by 90 degrees.

In the above table, for each of the elements - resistor, capacitor, and inductor - there is a phasor diagram. The diagram consists of two vectors rotating counterclockwise. The red and blue vectors represent the maximum magnitudes of the voltage and current respectively, for a given angle.

We can find the impedence if we look at the three circuit components from the table on one set of axes.



In the diagram, V_{Lmax}, represents the maximum voltage across the coil, V_{Cmax} the maximum voltage across the capacitor, V_{Rmax} the maximum voltage across the resistor, and V_{max} the maximum voltage across the circuit.

We also let V_L, V_C, and V_R be the rms voltages across the inductor, capacitor, and resistor, respectively, and V rms voltage across the circuit.

Using the above diagram and the Pythagorean Theorem, we can write

$$V_{max}^2 = V_{R_{max}}^2 + (V_{L_{max}} - V_{C_{max}})^2$$

or

Next recall, that $V_{rms} = \frac{V_{max}}{\sqrt{2}}$ or $V_{max} = \sqrt{2}V_{rms}$. It follows that

$$(\sqrt{2}V)^2 = (\sqrt{2}V_R)^2 + (\sqrt{2}V_L - \sqrt{2}V_C)^2.$$

Simplifying we get

$$V = \sqrt{V_R^2 + (V_L - V_C)^2}$$

or better yet,

$$V = \sqrt{(iR)^2 + (iX_{L,} - iX_C)^2}$$, where i is the rms current.

Which becomes

$$V = i\sqrt{R^2 + (X_L - X_C})^2$$

or

$$V_{rms} = i_{rms}\sqrt{R^2 + (X_{L,} - X_C})^2$$

Next, let Z the impedence, the overall resistance in the circuit. Z is given by

$$Z = \sqrt{R^2 + (X_{L,} - X_C})^2$$

and if we want, we can write

$$V_{rms} = i_{rms}Z$$

It can be shown that ϕ, the angle between the maximum voltage and the maximum current, is given by

$$\phi = tan^{-1}(\tfrac{X_L - X_C}{R})$$

If $\phi > 0,$ then voltage leads the current by ϕ degrees. This kind of current is called an ***inductive circuit***. If $\phi < 0$, then voltage lags the current by ϕ degrees. This kind of circuit is called a ***capacitive circuit.***

Problem

Analyze a series RLC circuit for which $R = 250\,\Omega,\ L = 0.600\,\text{H},\ C = 3.50$ µF, $f = 60\,\text{Hz}$, and $V_{rms} = 150$ V.

Solution

The reactances are given by $X_L = 2\pi fL = 226\ \Omega$ and $X_C = \frac{1}{2\pi fC} = 758\,\Omega.$
Therefore, the impedance is given by
$Z = \sqrt{R^2 + (X_L - X_C)^2} = \sqrt{(250\,\Omega)^2 - (226\,\Omega - 758\,\Omega)^2} = 588\,\Omega$

The rms current is given by $i_{rms} = \frac{V_{rms}}{Z} = \frac{150\,V}{588\,\Omega} = 0.255\,\text{A}$

The phase angle between the current and the angle is

$$\phi = tan^{-1}\left(\frac{X_L - X_C}{R}\right) = tan^{-1}\left(\frac{226\,\Omega - 758\,\Omega}{250\,\Omega}\right) = -64.8°$$

Since $\phi < 0$, this is a capacitive circuit where voltage lags the current.

The rms voltages across the elements are

$$V_R = i_{rms}R = (0.255\,\text{A})(250\,\Omega) = 63.8\,\text{V}$$
$$V_L = i_{rms}X_L = (0.255\,\text{A})(226\,\Omega) = 57.6\,\text{V}$$
$$V_C = i_{rms}X_C = (0.255\,\text{A})(758\,\Omega) = 193\,\text{V}$$

Note that the sum of the three rms voltages is 314 V, which is much greater than rms voltage of the generator, which is 150 V. The sum 314 V is a meaningless quantity, because when alternating voltages are added, both their amplitude and their phases must be taken into account. It's as meaningless as adding three vectors, with different directions, by just adding their magnitudes.

For information and formulas on power dissipated in an ac circuit, resonance in RLC series circuits and on transformers, for some of the upcoming problems, see lecture notes.

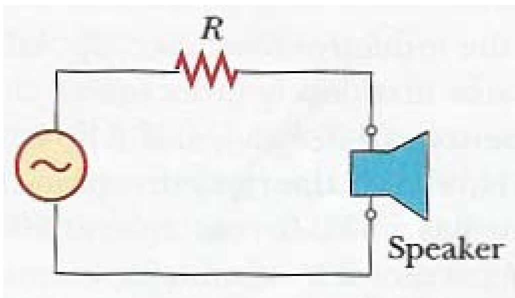
Exercises

1. An rms voltage of 100 V is applied to a purely resistive load of 5.0 Ω. Find (a) the maximum voltage applied, (b) the rms current supplied, (c) the maximum current applied, and (d) the power dissipated.

2. An ac voltage source has an output of $v = 150 sin 377t$. Find (a) the rms voltage output, (b) the frequency of the source, (c) the voltage at $t = 1/120\, s$. (d) find the maximum current in the circuit when the generator is connected to a 50 Ω resistor.

3. (a) What is the resistance of a lightbulb that uses a power of 75 W when connected to a 60 Hz power source with a peak voltage of 170 V? (b) What is the resistance of a 100 W bulb.

4. An audio amplifier, represented by the ac source and resisiter R in the figure below, delivers alternating voltages at audio frequencies to the speaker. If the source puts out an alternating voltage of 15 V (rms), resistance $R = 8.20\ \Omega$, and the speaker is equivalent to a resistance of 10.4 Ω, what is the time averaged power input to the speaker? (Hint: This is just another way of asking for the power delivered to the speaker.



5. When a 4.0 μF capacitor is connected to a generator whose rms output is 30 V, current in the circuit is observed to be 0.30 A. What is the frequency of the source?

7. A certain capacitor in a circuit has a capacitive reactance of 30 Ω when the frequency is 120 Hz. What capacitive reactance does the capacitor have at 10,000 Hz?

8. What value of capacitor must be inserted in a 60 Hz circuit in series with a generator of 170 V maximum output voltage to produce an rms current output of 0.75 A?

9. Show that the inductive reactance, X_L, has SI units of ohms.

10. The generator in an inductive ac circuit has an angular frequency of 120π rad/s. If $V_{max} = 140\,\mathrm{V}$ and $L = 0.100$ H, what is the rms current in the circuit?

11. An inductor has a 54 Ω reactance at 60 Hz. What will be the peak current if this inductor is connected to a 50 Hz source that produces a 100 V rms voltage.

12. A 20 mH inductor is connected in series with a 20 Ω resistor and a 60 Hz, 100 V rms source. Find (a) the rms current in th circuit, (b) the voltage drop across the inductor, (c) the voltage drop across the resistor, and (d) the phase angle for this circuit. (e) sketch the phasor diagram for this circuit.

13. A 10 μF capacitor and a 2 H inductor are connected in series with a 60 Hz source whose rms output is 100 V. Find (a) the rms current in the circuit, (b) the voltage drop across the inductor, (c) the voltage drop across the capacitor, and (d) the phase angle for the circuit. (e) Sketch the phasor diagram for this circuit.

14. A resistor $(R = 900\,\Omega)$, a capacitor $(C = 0.25\,\mu F)$, and an inductor $(L = 2.5\,H)$ are connected in series across a 240 Hz ac source for which $V_m = 140\,\mathrm{V}$. Calculate the (a) the impedence of the circuit, (b) peak current delivered by the source, and (c) phase and between current and voltage. (d) Is the current leading or lagging behind the voltage?

15. A 50 Ω resistor is connected in series with a 15 μF capacitor and a 60 Hz 120 V (rms) source. (a) Find the rmscurrent in th circuit. (b) What is the value of the inductor that must be inserted in the circuit to reduce the current to one-half the value found in part (a)?

16. A multimeter in an *RL* circuit records an rms current of 0.5 A and a 60 Hz rms generator voltage of 104 V. The average thermal power developed in the resistor is 10 W. Determine (a) the impedance of the circuit, (b) the resistance, R, and (c) the inductance, L.

17. An inductor and a resistor are connected in series. When connected to a 60 Hz, 90 V (rms) source, the voltage drop across the resistor is found to be 50 V and the power dissipated in the circuit is 14 W. Find (a) the value of the resistance and (b) the value of the inductance.

18. An ac voltage with an amplitude of 100 V is applied to a series combination of a 200 μF capacitor, a 100 mH inductor, and a 20 Ω resistor. Calculate the power factor for a frequency of 60 Hz.

19. An RLC circuit is used to tune a radio to an FM station broadcasting at 88.9 MHz. The resistance of the circuit 12 Ω and the capacitance is 1.4 pF. What inductance should be present in the circuit?

20. A resonant circuit in a radio receiver is tuned to a certain station when the inductor has a value of 0.2 mH and the capacitor has a value of 30 pF. Find the frequency of the radio station and the wavelength sent out by the station.

21. The AM band extends from approximately 500 kHz to 1600 kH. If a 2 μH inductor is used in tuning a circuit for a radio, what are the extremes that a capacitor must reach in order to cover the complete band of frequencies?

22. A series circuit contains a 3 H inductor, a 3 μF capacitor, and a 30 Ω resistor connected to a 120 V rms source of variable frequency. Find the power delivered to the circuit when the frequency of the source is (a) the resonance frequency; (b) one half the resonance frequency; (c) one fourth the resonance frequency; (d) two times the resonance frequency; (e) four times the resonant frequency. From your calculations, can you draw a conclusion about the frequency at which the maximum power is delivered to the circuit?

Transformers

23. A step-up transformer is designed to have an output voltage of 2200 V when the primary is connected across a 110 V source.

    a. If there are 80 turns on the primary winding, how many turns are required on the secondary?

    b. If a load resistor across the secondary draws a current of 1.5 A, what is the current in the primary?

24. At a given moment, every inhabitant of a city of 20,000 people turns on a 100 W lightbulb. Assume that no other power in the city is being used.

    a. If the utility company furnishes this power at 120 V, calculate the current in the power lines from the utility to the city.

    b. Calculate this current if the power company first steps up the voltage to 200,000 V.

    c. How much heat is lost in each one meter length of the power lines if the resistance of the lines is 5.00×10^{-4} Ω/m. Find this value for both parts a & b.

    d. If an individual line from the utility can handle only 100 A, how many lines are required to handle the current in each situation described?

25. A transformer on a pole near a factory steps the voltage down from 3600 V to 120 V. The transformer is to deliver 1000 kW to the factory at 90% efficiency. Find

a. the power delivered to the primary.

b. the current in the primary.

c. the current in the secondary.

26. An ac power generator produces 50 A at 3600 V. The voltage is stepped up to 100,000 V by a transformer, and the energy is transmitted through a long distance power line that has a resistance of 100 Ω. What percentage of the power delivered by the generator is dissipated as heat in the power lines?